$$t_{\text{grow}} \gtrsim 0.45 \frac{\eta}{1-\eta} \ln \left(\frac{M_{\text{BH}}}{M_{\text{BH},0}} \right) \text{ Gyr}$$